A New Mechanical Model of Magnetohydrodynamic Instabilities in Aluminium Electrolysis Cells

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Abstract

Understanding the magnetohydrodynamic metal pad instability (MPI) is important for aluminium electrolysis cells to operate stably and at a lower anode-to-cathode distance (ACD). Modelling the instability analytically requires solving a complex coupled system of fluids and electromagnetic equations. Pure analytical techniques are limited by the simplifying assumptions that make the equations solvable, but less representative of a real cell. Numerical simulations accounting for the great complexity of magnetohydrodynamics are much better at modelling a real cell, but often are expensive and can be time consuming. Another approach is using a simple mechanical model of the MPI, such as the compound pendulum given in [1] and expanded to include an oscillating current in [2]. This comparatively simple model offers great physical insight into the MPI and its mechanism but makes incorrect predictions about the parameter values that cause instability, when compared to real cells. In this work, we present a new mechanical model of the MPI in the form of a mobile with a variable center of mass. This new model shows instability at realistic cell parameters.

Keywords: Magnetohydrodynamic instability, Aluminium cell modelling

1. Introduction

Aluminium (Al) is produced using the Hall-Héroult process in Al electrolysis cells, where a molten cryolite layer floats atop a molten Al layer and both fluid layers are situated in between a carbon anode at the top and a carbon cathode at the bottom [3]. Alumina is dissolved in the cryolite, and a vertical electric current passes from the anode through the fluid layers to the cathode, decomposing the alumina and producing Al [4]. Due to the cryolite's very low electrical conductivity (~233 S/m), a lot of the electrical energy is transformed into heat by Joule heating [5] and produces no Al. Reducing the cryolite thickness, quantified by the anode to cathode distance (ACD), would be an effective way to reduce the energy loss, but reducing it beyond a critical threshold makes the Al cell unstable [3].

Natural disturbances exist on the cryolite-Al interface and can be thought of as a sum of interfacial wave modes that are decoupled [1]. When there is no current in the system and consequently there are no electromagnetic forces, the (purely gravitational) interfacial waves die out due to viscous damping. However, when current is present, electromagnetic forces are produced by the interaction between current and induced magnetic field. Horizontal electromagnetic forces may amplify waves present on the cryolite-Al interface [4], causing a circulating wave that can grow until the Al shorts to the anode [3]. The instability is magnetohydrodynamic (MHD) in nature, known as the metal pad instability (MPI), where the electromagnetic forces - couple interfacial gravitational wave modes having close frequencies leading to instability [1, 6]

A mechanical model of the MPI was given in [1] and then used again in [2]. The model is a compound pendulum, shown in Figure 1. It consists of a broad and thin Al plate connected by a rigid massless strut to a fixed ceiling. The Al plate can swing about the two horizontal axes (x and y axes), with the pivot at the connection point between the strut and the fixed ceiling. The gap between the Al plate and the fixed ceiling is filled with frictionless cryolite creating a path for current to pass from the "anode" ceiling to the Al plate.

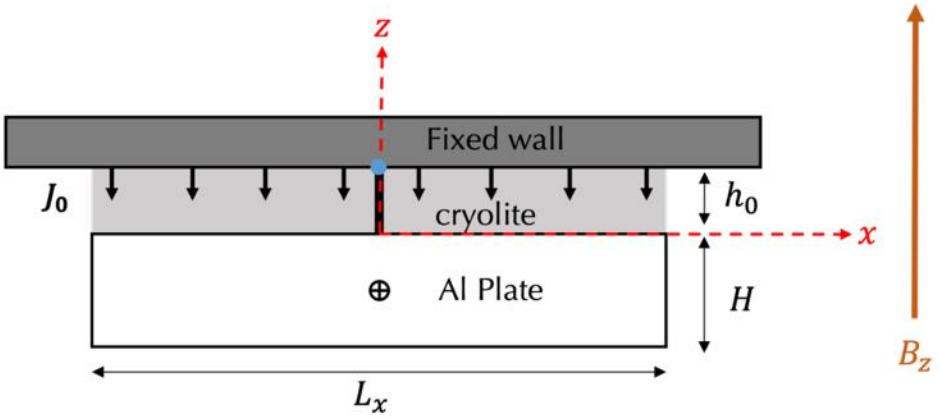

Figure 1. Pendulum model of the MPI used in [2] at equilibrium state. The Al plate's center of mass (COM) is half-way through its thickness and is marked by a circle with a cross. The pivot point is indicated by a blue disc.

where:

h_0	Thickness of the cryolite or ACD, m
H	Thickness of the Al plate (analogous to metal pad height), m
L_x	Length of the Al plate in the x direction, m
J_0	Nominal current density in bath, A/m^2
B_z	Vertical constant and uniform magnetic flux density, T.

If there is no current and we tilt the Al plate by a small amount (perturb it) from its equilibrium position (shown in Figure 2), gravity acts as a restoring force pushing the Al plate back towards the equilibrium position. This makes the Al plate oscillate about the equilibrium position either indefinitely when friction is neglected, or till the Al plate rests at the equilibrium position when friction is considered. The Al plate oscillates at its natural gravitational frequency in each horizontal direction, and those frequencies are [2]:

$$\omega_x^2 = \frac{12g\left(h_0+\frac{H}{2}\right)}{L_y^2} \tag{1}$$

$$\omega_y^2 = \frac{12g\left(h_0+\frac{H}{2}\right)}{L_x^2} \tag{2}$$

where:

ω_x	Natural gravitational frequencies of the Al plate in the x direction, rad/s
ω_y	Natural gravitational frequencies of the Al plate in the y direction, rad/s
L_y	Width of the Al plate in the y direction, m
g	Gravitational acceleration, 9.81 m/s^2.

Thus, when there is no current, the Al plate motion in the x and y directions is decoupled. However, when current is present, it interacts with the imposed uniform vertical magnetic flux motion which was of the same order of magnitude as the one reported in [3] from a simulation of the TRIMET cell done in MHD-Valdis. Although not exact, the results are promising and could get closer with better calibration of the damping parameter ζ. The mobile model presents an opportunity to quickly explore the stability of the cell in a parameter space. More specifically, rather than running many computationally expensive numerical magnetohydrodynamic simulations to see if the Al cell is stable for multiple combinations of parameters, one can first check the stability of the mobile model of that cell by solving its equations of motion with the same combinations of parameters. This has the benefit of rapidly narrowing down the combinations of cell parameters to be simulated numerically, reducing the computational expense. The mobile model would be a complement tool to numerical magnetohydrodynamic simulations.

The mobile model is an improvement over the previous pendulum model since it shows stability/instability at realistic cell parameters. However, it is still missing important parameters and features that impact the magnetohydrodynamic stability of a realistic cell. For example, an actual Al cell is subjected to a vertical magnetic flux density generated by the adjacent pot rows, internal conductors, external conductors, and magnetization effects. Thus, the vertical magnetic flux density has a distribution in space – $B_z(x, y)$ – unlike the mobile model where it is assumed to be uniform. This could a potential avenue for improving the model.

Moving forward, we plan on tuning the model more so it can better capture the growth/decay rates of interfacial waves from an actual cell. Also, we plan on leveraging the mobile model to explore the phase space of the dynamic stabilization of the cell [3, 7]. In particular, we are interested in looking at the stabilization effect of adding an oscillating vertical magnetic flux density at different oscillation frequencies and amplitudes.

5. References

1. P. A. Davidson and R. I. Lindsay, Stability of interfacial waves in aluminium reduction cells, *J. Fluid Mech.* 362, 1998, 273-295.
2. Ibrahim Mohammad and Douglas H. Kelley, Stabilizing a low dimensional model of magnetohydrodynamic instabilities in aluminum electrolysis cells, *Light Metals* 2022, 512-519.
3. Ibrahim Mohammad et al., Oscillating currents stabilize aluminum cells for efficient, low carbon production, *JOM* 74, 2022, 1908-1915.
4. Sergei Molokov, Gennady El and Alexander Lukyanov, Classification of instability modes in a model of aluminium reduction cells with a uniform magnetic field, *Theoretical and Computational Fluid Dynamics* 25, 2011, 261-279.
5. Ibrahim Mohammad, *On stabilizing aluminium electrolysis cells with oscillating currents*, PhD Thesis, University of Rochester, New York, USA, 2023.
6. V. Bojarevics and M. V. Romerio, Long waves instability of liquid metal-electrolyte interface in aluminium electrolysis cells: a generalization of Sele's criterion, *Eur. J. Mech B/Fluids* 13, 1994, 33-56.
7. Marc Dupuis and Valdis Bojarevics, Stabilizing aluminium reduction cells by oscillating currents in magnetic compensation loops, *Proceedings of 40th International ICSOBA Conference*, Athens, Greece, 10 - 14 October 2022, Paper AL 20, *TRAVAUX* 51, 1247-1258.
8. Swaroop K. Yalla and Ahsan Kareem, Beat phenomenon in combined structure-liquid damper system, *Engineering Structures* 23, 2001, 622-630.
9. S. Graham Kelly, *Mechanical vibrations: theory and applications*, SI Edition, Dusseldorf, CENGAGE Learning, 2012, 853 pages.
10. Marc Dupuis and Michaël Pagé, Modelling gravity waves in an aluminium reduction cell using OpenFoam, *INTERNATIONAL ALUMINIUM JOURNAL*, 1-2/2016, 58-61.